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Medium-modified Casimir forces

M S Tomaš

Rudjer Bošković Institute, PO Box 180, 10002 Zagreb, Croatia

E-mail: tomas@thphys.irb.hr

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Abstract
We argue that the results for the vacuum forces on a slab and on an atom embedded in a magnetodielectric medium near a mirror, obtained using a recently suggested Lorentz-force approach to the Casimir effect, are equivalent to the corresponding results obtained in a traditional way. We also derive a general expression for the atom–atom force in a medium and extend a few classical results concerning this force in vacuum and dielectrics to magnetodielectric systems. This, for example, reveals that the (repulsive) interaction between atoms of different polarizability type is at small distances unaffected by a (weakly polarizable) medium.

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Although modifications of the Casimir and van der Waals forces due to the presence of a medium between the interacting objects were the subject of interest for a long time [1–5], this issue is still of great importance owing to the dominant role of these forces at small distances and to the rapid progress in micro and nanotechnologies. A common way of extending the Lifshitz theory [6] of the Casimir effect [7] to material cavities is to (eventually) employ the Minkowski stress tensor $T_{ij} = \frac{1}{4\pi}\langle D_i E_j + H_i B_j - \frac{1}{2}(\mathbf{D}\cdot\mathbf{E} + \mathbf{H}\cdot\mathbf{B})\delta_{ij}\rangle$ when calculating the force [1, 2, 4, 8]. Recently, however, a Lorentz-force approach to the Casimir effect was suggested [9] (see also [10]) in which the relevant stress tensor is of the form (the brackets denote the average with respect to fluctuations, and we use the standard notation for the macroscopic field operators)

$$T_{ij}^{(L)} = T_{ij} - \langle P_i E_j - M_i B_j - \tfrac{1}{2}(\mathbf{P}\cdot\mathbf{E} - \mathbf{M}\cdot\mathbf{B})\delta_{ij}\rangle. \qquad (1)$$

In this work, we reinterpret the results for the vacuum forces on a slab and on an atom embedded in a semi-infinite magnetodielectric cavity (see figure 1) as recently obtained using $T_{ij}^{(L)}$ [11, 12] and argue their equivalence to the corresponding results obtained using T_{ij}. Also, we derive a general expression for the atom–atom force in a medium and extend a few well-known results for this force to magnetodielectric systems.

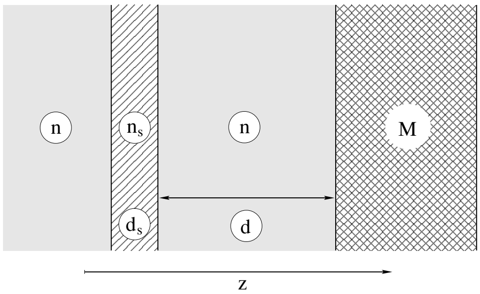

Figure 1. A slab in front of a mirror shown schematically. The (complex) refraction index of the slab is $n_s(\omega) = \sqrt{\varepsilon_s(\omega)\mu_s(\omega)}$ and that of the surrounding medium is $n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)}$. The mirror is described by its reflection coefficients $R^q(\omega, k)$, with k being the in-plane wave vector of a wave. The arrow indicates the direction of the force on the slab.

When calculating $T_{ij}^{(L)}$ for planar geometry [9], the zero-temperature force on the slab per unit area in the configuration of figure 1 can be written as [11]

$$f^{(L)}(d) = f(d) + \tilde{f}(d), \tag{2a}$$

$$f(d) = \frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}k\, k\kappa \sum_{q=p,s}\left[\frac{1}{(r^q R^q\, \mathrm{e}^{-2\kappa d})^{-1} - 1}\right](\mathrm{i}\xi, k), \tag{2b}$$

$$\tilde{f}(d) = \frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty \mathrm{d}k\, k\kappa \sum_{q=p,s}\left[\frac{(\varepsilon^{-1}-1)\delta_{qp} + (\mu - 1)\delta_{qs}}{(r^q R^q\, \mathrm{e}^{-2\kappa d})^{-1} - 1}\right](\mathrm{i}\xi, k)$$
$$+ \frac{\hbar}{8\pi^2 c^2}\int_0^\infty \mathrm{d}\xi\, \xi^2 \mu(n^2 - 1)\int_0^\infty \frac{\mathrm{d}k\, k}{\kappa}\sum_{q=p,s}\Delta_q\left[\frac{(1+r^q)^2 - t^{q2}}{(R^q\, \mathrm{e}^{-2\kappa d})^{-1} - r^q}\right](\mathrm{i}\xi, k), \tag{2c}$$

corresponding to the decomposition of the stress tensor in (1). Here $\kappa(\xi, k) = \sqrt{n^2(\mathrm{i}\xi)\xi^2/c^2 + k^2}$ is the perpendicular wave vector in the cavity at the imaginary frequency, $\Delta_q = \delta_{qp} - \delta_{qs}$, r^q and t^q are Fresnel coefficients for the (symmetrically bounded) slab and R^q are those for the mirror. As seen, the first term in $\tilde{f}$ may be combined with the traditional (Minkowski) force f to form a medium-screened Casimir force, with the contributions of TM and TE polarized waves scaled, respectively, by $1/\varepsilon$ and μ, whereas the remaining term in $\tilde{f}$ may be regarded as a medium-assisted force [11]. However, owing to this (additional) screening of the force, it turns out that the medium-screened van der Waals and Casimir forces have a rather unusual dependence on the material parameters of the system [12] and, therefore, this formal combination demands reconsideration. Here, we interpret (2) by recalling that, according to the $T \neq 0$ quantum-field-theoretical approach to the Casimir force, the Minkowski stress tensor T_{ij} corresponds to the effective stress tensor in a medium which is in mechanical equilibrium [1, 4, 13]. If so, the (unbalanced) second term in (1), which leads to $\tilde{f}$, gives a force on the medium. Therefore, the force on the slab is given solely by the traditional force f in (2), whereas $\tilde{f}$ describes a force on the medium. Indeed, note that $\tilde{f}$ vanishes when there is no medium, $\varepsilon = \mu = 1$, and is nonzero when there is no slab $n_s = n$ ($r^q = 0$ and $t^q = \mathrm{e}^{-\kappa d_s}$ in (2c)), in this case clearly representing the force on a layer of the medium.

The force on an atom near a mirror can be obtained from (2) by assuming that the slab consists of a thin ($d_s \ll d$) layer of the cavity medium with a small number of atoms A

embedded in it [12]. Then, from the Lorentz–Lorenz (Clausius–Mossotti) relation [14], it follows that

$$\varepsilon_s = \varepsilon + 4\pi N_A \alpha_e^A, \qquad \alpha_e^A = \alpha_{e0}^A \left(\frac{\varepsilon+2}{3}\right)^2 \left[1 + \mathcal{O}\left(N_A \alpha_{e0}^A\right)\right], \tag{3}$$

where N_A is the number density and α_{e0}^A is the electric vacuum polarizability of embedded atoms. Assuming a similar relation between μ_s, μ and the effective atomic magnetic polarizability α_m^A, for the force on an *embedded* atom at distance d from the mirror we find (for details, see the derivation of equation (14) in [12][1])

$$f_A^{(L)}(d) = f_A(d) + \tilde{f}_A(d), \tag{4a}$$

$$f_A(d) = \frac{\hbar}{\pi c^2} \int_0^\infty \mathrm{d}\xi\, \xi^2 \int_0^\infty \mathrm{d}k\, k\, \mathrm{e}^{-2\kappa d} \left\{ \left[\alpha_e^A \left(2\frac{\kappa^2 c^2}{\varepsilon \xi^2} - \mu\right) - \alpha_m^A \varepsilon\right] R^p \right.$$
$$\left. + \left[\alpha_m^A \left(2\frac{\kappa^2 c^2}{\mu \xi^2} - \varepsilon\right) - \alpha_e^A \mu\right] R^s \right\} (\mathrm{i}\xi, k), \tag{4b}$$

$$\tilde{f}_A(d) = \frac{\hbar}{\pi c^2} \int_0^\infty \mathrm{d}\xi\, \xi^2 \int_0^\infty \mathrm{d}k\, k\, \mathrm{e}^{-2\kappa d} \left\{ \left(\frac{1}{\varepsilon} - 1\right) \left[\alpha_e \left(2\frac{\kappa^2 c^2}{\varepsilon \xi^2} - \mu\right) - \alpha_m \varepsilon\right] R^p \right.$$
$$\left. + (\mu - 1) \left[\alpha_m \left(2\frac{\kappa^2 c^2}{\mu \xi^2} - \varepsilon\right) - \alpha_e \mu\right] R^s + \left(\mu - \frac{1}{\varepsilon}\right) \left[\alpha_e \mu R^p - \alpha_m \varepsilon R^s\right] \right\} (\mathrm{i}\xi, k). \tag{4c}$$

As before, this can be combined to give a medium-screened and a medium-assisted force on the atom [12]. However, according to the interpretation adopted here, the true force on the atom is f_A, whereas $\tilde{f}_A$ may be regarded as an atom-induced force on the medium (per atom). We note that (4*b*) extends (in different directions) previous results for the atom–mirror force under various circumstances [2, 3, 15–17] by fully accounting for the magnetic properties of the system.

Equation (4*b*) enables one to calculate the force between two atoms in a magnetodielectric medium. This force can be found by assuming a single-medium mirror consisting of the cavity medium with a small number of, say, type B atoms embedded in it, so that $\varepsilon_m = \varepsilon + 4\pi N_B \alpha_e^B$ and $\mu_m = \mu + 4\pi N_B \alpha_m^B$. In this case, the reflection coefficients of the mirror can be approximated by [12]

$$R^p(\mathrm{i}\xi, k) \simeq \frac{\pi N_B \xi^2}{\kappa^2 c^2} \left[\alpha_e^B \mu \left(\frac{2\kappa^2 c^2}{n^2 \xi^2} - 1\right) - \alpha_m^B \varepsilon\right], \tag{5}$$

and $R^s = R^p\left(\varepsilon \leftrightarrow \mu, \alpha_e^B \leftrightarrow \alpha_m^B\right)$. Also, the potential energy of the atom $U_A(d) = -\int_\infty^d \mathrm{d}l\, f_A(l)$ is given by $U_A(d) = N_B \int_{z \geqslant d} \mathrm{d}^3\mathbf{r}\, U_{AB}(r)$, where $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$ and $U_{AB}(r)$ is the interaction energy between the atoms A and B. Using the identity $\exp(-2\kappa d) = (2\kappa^2/\pi) \int_{z \geqslant d} \mathrm{d}^3\mathbf{r} \exp(-2\kappa r)/r$ and combining these relations, from (4*b*) and (5) we obtain the extension of the Feinberg and Sucher formula [19] to material systems

$$U_{AB}(r) = \frac{\hbar}{16\pi r^6} \int_0^\infty \mathrm{d}\xi\, \mathrm{e}^{-2n\xi r/c} F\left(\frac{2n\xi r}{c}\right) \left(\frac{\alpha_e^A \alpha_e^B}{\varepsilon^2} + \frac{\alpha_m^A \alpha_m^B}{\mu^2}\right)$$
$$- \frac{\hbar}{4\pi c^2 r^4} \int_0^\infty \mathrm{d}\xi\, \xi^2 \mathrm{e}^{-2n\xi r/c} G\left(\frac{2n\xi r}{c}\right) \left(\alpha_e^A \alpha_m^B + \alpha_m^A \alpha_e^B\right), \tag{6}$$

where $F(x) = x^4 + 4x^3 + 20x^2 + 48x + 48$ and $G(x) = (x+2)^2$.

[1] Owing to a lapse, the replacements $\alpha_e \to \alpha_e/\varepsilon$ and $\alpha_m \to \alpha_m/\mu$ must be made in the last term of this equation.

At small distances between the atoms, $r \ll c/\Omega$ (Ω is an effective upper limit in the integration over ξ [17]), we may let $\exp(-2n\xi r/c) \simeq 1$, $F(2n\xi r/c) \simeq 48$ and $G(2n\xi r/c) \simeq 4$ in (6). This gives for the atom–atom force $f_{AB} = -\mathrm{d}U_{AB}/\mathrm{d}r$ at small distances

$$f_{AB}(r) = \frac{18\hbar}{\pi r^7}\int_0^\infty \mathrm{d}\xi \left(\frac{\alpha_e^A\alpha_e^B}{\varepsilon^2} + \frac{\alpha_m^A\alpha_m^B}{\mu^2}\right) - \frac{4\hbar}{\pi c^2 r^5}\int_0^\infty \mathrm{d}\xi\, \xi^2\left(\alpha_e^A\alpha_m^B + \alpha_m^A\alpha_e^B\right), \tag{7}$$

which generalizes the well-known results for the van der Waals–London force [1, 18, 19]. We observe that whereas the force between the atoms of the same polarizability type is strongly affected by the surrounding medium, the (repulsive) force between the atoms of different polarizability type (in weakly polarizable media) remains the same as in vacuum. At large distances between the atoms, the main contribution to the integral in (6) comes from the $\xi \simeq 0$ region. Approximating the ξ-dependent quantities by their static values (denoted by the subscript 0) and then performing the integration, we arrive at

$$f_{AB}(r) = \frac{7\hbar c}{4\pi n_0^5 r^8}\left[23\left(\alpha_e^A\alpha_e^B\mu^2 + \alpha_m^A\alpha_m^B\varepsilon^2\right) - 7\left(\alpha_e^A\alpha_m^B + \alpha_m^A\alpha_e^B\right)\varepsilon\mu\right]_0, \tag{8}$$

which gives the medium corrections to the retarded atom–atom force [15, 19] and extends the previous considerations of these corrections [1, 4] to magnetodielectric systems. As seen, the repulsive component of the force is in a (weakly polarizable) medium simply scaled by n_0^{-3}, whereas the modification of its attractive part is more complex.

We end this short discussion by noting that the symmetry of (4*b*) under the transformation $\alpha_e \leftrightarrow \alpha_m, \varepsilon \leftrightarrow \mu$ and $R^p \leftrightarrow R^s$ (and consequently that of (6)–(8) with respect to the replacements $\alpha_e \leftrightarrow \alpha_m$ and $\varepsilon \leftrightarrow \mu$) is a consequence of the invariance of the Minkowski stress tensor T_{ij} with respect to a duality transformation, e.g. $\mathbf{D} \to \mathbf{B}, \mathbf{E} \to \mathbf{H}, \mathbf{B} \to -\mathbf{D}$ and $\mathbf{H} \to -\mathbf{E}$ [16]. This symmetry is lost in the second term of $T_{ij}^{(L)}$, so that the force $\tilde{f}_A$ is not invariant under the replacement of the electric and magnetic quantities. Since the medium-screened atom–mirror and atom–atom forces are combinations of the corresponding f's and $\tilde{f}$'s, this explains the unusual (asymmetric) medium effects on these forces found in [12].

In conclusion, when properly interpreted, the result for the vacuum force on an object (a slab or an atom) embedded in a medium near a mirror, obtained using the Lorentz-force approach to the Casimir effect, agrees with that obtained in a traditional way. Extensions of the well-known results for the atom–atom force to magnetodielectric systems reveal that the (repulsive) interaction between atoms of different polarizability type remains (in weakly polarizable media) the same as in vacuum at small distances and is scaled by n_0^{-3} at large distances. Medium corrections to the force between atoms of the same polarizability type are more complex and depend on the polarizability type of atoms.

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